Practice Problems

1. A compact car, mass 725 kg, is moving at +100 km/h.
   a. Find its momentum.

\[ p = \frac{mv}{100 \text{ km/h}} = \frac{725 \text{ kg}(27.8 \text{ m/s})}{27.8 \text{ m/s}} = 2.02 \times 10^4 \text{ kg} \cdot \text{m/s} \]

b. At what velocity is the momentum of a larger car, mass 2175 kg, equal to that of the smaller car?

\[ v = \frac{p}{m} = \frac{2.02 \times 10^4 \text{ kg} \cdot \text{m/s}}{2175 \text{ kg}} = 9.29 \text{ m/s} = 33.4 \text{ km/h} \]

2. A snowmobile has a mass of \(2.50 \times 10^2\) kg. A constant force is exerted on it for 60.0 s. The snowmobile's initial velocity is 6.00 m/s and its final velocity 28.0 m/s.
   a. What is its change in momentum?

\[ \Delta p = m(v_f - v_i) = (250 \text{ kg})(28.0 \text{ m/s} - 6.0 \text{ m/s}) = 5.50 \times 10^3 \text{ kg} \cdot \text{m/s} \]

b. What is the magnitude of the force exerted on it?

\[ F = \frac{\Delta p}{\Delta t} = \frac{5.50 \times 10^3 \text{ kg} \cdot \text{m/s}}{60.0 \text{ s}} = 91.7 \text{ N} \]

3. The brakes exert a \(6.40 \times 10^2\) N force on a car weighing 15 680 N and moving at 20.0 m/s. The car finally stops.
   a. What is the car's mass.

\[ m = \frac{W}{g} = \frac{15 680 \text{ N}}{9.80 \text{ m/s}^2} = 1.60 \times 10^3 \text{ kg} \]

Practice Problems

b. What is its initial momentum?

\[ p_i = mv_i = (1600 \text{ kg})(20.0 \text{ m/s}) = 3.20 \times 10^4 \text{ kg} \cdot \text{m/s} \]

c. What is the change in the car's momentum?

\[ \Delta p = p_f - p_i = 0 - 3.20 \times 10^4 \text{ kg} \cdot \text{m/s} = -3.20 \times 10^4 \text{ kg} \cdot \text{m/s} \]

d. How long does the braking force act on the car to bring it to a halt?

\[ F\Delta t = \Delta p, \quad \Delta t = \frac{\Delta p}{F} = \frac{3.20 \times 10^4 \text{ kg} \cdot \text{m/s}}{(-6.40 \times 10^2 \text{ N})} = 50.0 \text{ s} \]

4. Figure 9-1 shows, as a function of time, the force exerted by a ball that collided with a box at rest. The impulse, \(F\Delta t\), is the area under the curve.
   a. Find the impulse given to the box by the bell.

\[ F\Delta t = \text{Area} = (52.5 \text{ squares})(0.100 \text{ N} \cdot \text{s/square}) = 5.25 \text{ N} \cdot \text{s} \]

b. If the box has a mass of 2.4 kg, what velocity did it have after the collision?

\[ \Delta p = m\Delta v \quad \Delta p = F\Delta t, \quad \frac{\Delta p}{m} = \frac{F\Delta t}{m} = \frac{(5.25 \text{ N} \cdot \text{s})}{(2.4 \text{ k} \cdot \text{s})} = 2.2 \text{ m/s} \]
5. A 0.105-kg hockey puck moving at 48 m/s is caught by a 75-kg goalie at rest. With what speed does the goalie slide on the ice?

\[ p_b + p_s = p_b' + p_s' \text{ or } m_b v_b + m_s v_s = m_b v_b' + m_s v_s' \]

Since \( v_s = 0 \), \( m_b v_b = (m_b + m_s) v' \)

where \( v' = v_b' = v_s' \) is the common final speed of goalie and puck.

\[ v' = \frac{m_b v_b}{m_b + m_s} \]

\[ = \frac{(0.105 \text{ kg})(48 \text{ m/s})}{(0.105 \text{ kg} + 75 \text{ kg})} \]

\[ = 0.067 \text{ m/s} \]

6. A 35.0-g bullet strikes a 5.0-kg stationary wooden block and embeds itself in the block. The block and bullet fly off together at 8.6 m/s. What was the original velocity of the bullet?

\[ m_b v_b + m_w v_w = (m_b + m_w) v' \text{ where } v' \text{ is the common final velocity of bullet and wooden block.} \]

Since \( v_w = 0 \),

\[ v_b = \frac{(m_b + m_w) v'}{m_b} \]

\[ = \frac{(0.035 \text{ kg} + 5.0 \text{ kg})(8.6 \text{ m/s})}{(0.035 \text{ kg})} \]

\[ = 1.2 \times 10^3 \text{ m/s}. \]

7. A 35.0-g bullet moving at 475 m/s strikes a 2.5-kg wooden block. The bullet passes through the block, leaving at 275 m/s. The block was at rest when it was hit. How fast is it moving when the bullet leaves?

\[ m_b v_b + m_w v_w = m_b v_b' + m_w v_w' \text{ with } v_w = 0. \]

\[ v_w' = \frac{(m_b v_b - m_b v_b')}{m_w} = \frac{m_b (v_b - v_b')}{m_w} \]

\[ = \frac{(0.035 \text{ kg})(475 \text{ m/s} - 275 \text{ m/s})}{(2.5 \text{ kg})} \]

\[ = 2.8 \text{ m/s} \]

8. A 0.50-kg ball traveling at 6.0 m/s collides head-on with a 1.00-kg ball moving in the opposite direction at a velocity of -12.0 m/s. The 0.50-kg ball moves away at -14 m/s after the collision. Find the velocity of the second ball.

\[ m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \text{, so } v_B' \]

\[ = \frac{(m_A v_A + m_B v_B - m_A v_A')}{m_B} \]

\[ = \frac{((0.50 \text{ kg})(6.0 \text{ m/s}) + (1.00 \text{ kg})(-12.0 \text{ m/s})}{(0.50 \text{ kg})(-14 \text{ m/s})/(1.00 \text{ kg})} \]

\[ = -2 \text{ m/s} \]

9. A 4.00-kg model rocket is launched, shooting 50.0 g of burned fuel from its exhaust at an average velocity of 625 m/s. What is the velocity of the rocket after the fuel has burned? (Ignore effects of gravity and air resistance.)

\[ p_i + p_f = p_{i}' + p_{f}' \text{ where } p_i + p_f = 0. \]

If the initial mass of the rocket (including fuel) is \( m_i = 4.00 \text{ kg} \), then the final mass of the rocket is \( m_i' = 4.00 \text{ kg} - 0.050 \text{ kg} = 3.95 \text{ kg} \).

\[ 0 = m_i' v_i' + m_f v_f' \]

\[ v_i' = \frac{-m_f v_f'}{m_i'} \]

\[ = \frac{- (0.050 \text{ kg})(-625 \text{ m/s})}{(3.95 \text{ kg})} \]

\[ = 7.91 \text{ m/s} \]

10. A thread holds two carts together on a frictionless surface as in Figure 9-12. A compressed spring acts upon the carts. After the thread is burned, the 1.5-kg cart moves with a velocity of 27 cm/s to the left. What is the velocity of the 4.5-kg cart?

\[ p_A + p_B = p_A' + p_B' \text{ with } p_A + p_B = 0, \]

\[ m_B v_B = -m_A v_A' \]

so \( v_B = \frac{-m_A v_A'}{m_B} = \frac{-(1.5 \text{ kg})(-27 \text{ cm/s})}{(4.5 \text{ kg})} \)

\[ = 9.0 \text{ cm/s, or } 9.0 \text{ cm/s to the right}. \]
11. Two campers dock a canoe. One camper steps onto the dock. This camper has a mass of 80.0 kg and moves forward at 4.0 m/s. With what speed and direction do the canoe and the other camper move if their combined mass is 110 kg?

\[
p_A + p_B = p_A' + p_B' \quad \text{with} \quad p_A + p_B = 0, \\
\frac{m_A v_A}{m_B} = -\frac{m_B v_B}{m_B}, \quad \text{so} \\
v_B' = -\frac{m_A v_A}{m_B} \\
= -\frac{(80.0 \text{ kg})(4.0 \text{ m/s})}{(110 \text{ kg})} \\
= -2.9 \text{ m/s}, \text{ or } 2.9 \text{ m/s in the opposite direction.}
\]

12. A colonial gunner sets up his 225 kg cannon at the edge of the flat top of a high tower. It shoots a 4.5 kg cannon ball horizontally. The ball hits the ground 215 m from the base of the tower. The cannon also moves, on frictionless wheels, and falls off the back of the tower, landing on the ground.

a. What is the horizontal distance of the cannon's landing, measured from the base of the back of the tower?

Both the cannon and the ball fall to the ground in the same time from the same height. In that fall time, the ball moves 215 m, the cannon an unknown distance we will call \( x \). Now \( t = \frac{d}{v} \), so \( \frac{(215 \text{ m})}{v_{\text{ball}}} = \frac{x}{v_{\text{cannon}}} \), so \( x = \left[ \frac{v_{\text{cannon}}}{v_{\text{ball}}} \right] \) related by conservation of momentum;

\[
(4.5 \text{ kg})v_{\text{ball}} = -(225 \text{ kg})v_{\text{cannon}}, \quad \text{so} \\
\frac{v_{\text{cannon}}}{v_{\text{ball}}} = \left( \frac{4.5}{225} \right) \\
\text{Thus } x = \left( \frac{4.5}{225} \right)(215 \text{ m}) = 4.3 \text{ m.}
\]

b. Why do you not need to know the width of the tower?

While on top, the cannon moves with no friction, and its velocity doesn't change, so it can take any amount of time to reach the back edge.

13. A 1325-kg car moving north at 27.0 m/s collides with a 2165-kg car moving east at 17.0 m/s. They stick together. Draw a vector diagram of the collision. In what direction and with what speed do they move after the collision?

\[
p_N + p_E = p' (\text{vector sum}) \\
p_N = m_N v_N = (1325 \text{ kg})(27.0 \text{ m/s}) \\
= 3.58 \times 10^4 \text{ kg} \cdot \text{m/s} \\
p_E = m_E v_E = (2165 \text{ kg})(17.0 \text{ m/s}) \\
= 3.68 \times 10^4 \text{ kg} \cdot \text{m/s}
\]

\[
\tan \theta = \frac{p_N}{p_E} = \frac{3.58 \times 10^4 \text{ kg} \cdot \text{m/s}}{3.68 \times 10^4 \text{ kg} \cdot \text{m/s}} = 0.973, \\
\theta = 44.2^\circ, \text{ north of east} \\
(p')^2 = (p_N)^2 + (p_E)^2 \\
= (3.58 \times 10^4 \text{ kg} \cdot \text{m/s})^2 \\
+ (3.68 \times 10^4 \text{ kg} \cdot \text{m/s})^2 \\
= 2.64 \times 10^9 \text{ kg}^2 \cdot \text{m}^2/\text{s}^2, \\
p' = 5.13 \times 10^4 \text{ kg} \cdot \text{m/s} \\
v' = \frac{p'}{m_N + m_E} \\
= \frac{(5.13 \times 10^4 \text{ kg} \cdot \text{m/s})}{(1325 \text{ kg} + 2165 \text{ kg})} \\
= 14.7 \text{ m/s}
3. The hockey puck shot in the previous problem has a mass of 0.115 kg and was at rest before the shot. With what speed does it head toward the goal?

\[ F \Delta t = m \Delta v, \text{ so } \Delta v = \frac{F \Delta t}{m} = \frac{4.8 \text{ kg} \cdot \text{m/s}}{0.115 \text{ kg}} = 42 \text{ m/s} \]

4. A force of 6.00 N acts on a 3.00-kg object for 10.0 s.
   a. What is the object's change in momentum?
   \[ m \Delta v = F \Delta t = (6.00 \text{ N})(10.0 \text{ s}) = 60.0 \text{ N} \cdot \text{s} \]
   b. What is its change in velocity?
   \[ m \Delta v = F \Delta t, \text{ so } \Delta v = \frac{F \Delta t}{m} = \frac{60.0 \text{ N} \cdot \text{s}}{30.0 \text{ kg}} = 20.0 \text{ m/s} \]

5. The velocity of a 600 kg auto is changed from 10.0 m/s to 44.0 m/s in 68.0 s by an applied, constant force.
   a. What change in momentum does the force produce?
   \[ \Delta p = m \Delta v = (600 \text{ kg})(44.0 \text{ m/s} - 10.0 \text{ m/s}) = 2.04 \times 10^4 \text{ N} \cdot \text{s} \]
   b. What is the magnitude of the force?
   \[ F \Delta t = m \Delta v, \text{ so } F = \frac{m \Delta v}{\Delta t} = \frac{2.04 \times 10^4 \text{ N} \cdot \text{s}}{68.0 \text{ s}} = 300 \text{ N} \]

6. A 845 kg drag race car accelerates from rest to 100 km/h in 0.90 seconds.
   a. What is the change in momentum of the car?
   \[ m \Delta v = (845 \text{ kg}) \left[ \left( \frac{100 \text{ km}}{1 \text{ h}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) - 0 \right] = 2.35 \times 10^4 \text{ kg} \cdot \text{m/s} \]

b. What average force is exerted on the car?
   \[ F \Delta t = m \Delta v, \text{ so } F = \frac{m \Delta v}{\Delta t} = \frac{2.35 \times 10^4 \text{ kg} \cdot \text{m/s}}{0.90 \text{ s}} = 2.6 \times 10^4 \text{ N} \]

7. A sprinter with a mass of 76 kg accelerates from 0 to 9.4 m/s in 2.8 s. Find the average force acting on the runner.
   \[ m \Delta v = (76 \text{ kg})(9.4 \text{ m/s} - 0) = 7.1 \times 10^2 \text{ kg} \cdot \text{m/s} \]
   \[ F \Delta t = m \Delta v, \text{ so } F = \frac{m \Delta v}{\Delta t} = \frac{7.1 \times 10^2 \text{ kg} \cdot \text{m/s}}{2.8 \text{ s}} = 2.6 \times 10^2 \text{ N} \]

8. A 0.25 kg soccer ball is rolling 6.0 m/s toward a player. The player kicks the ball back in the opposite direction and gives it a velocity of -14 m/s. What is the average force during the interaction between the player's foot and the ball if the interaction lasts 2.0 x 10^2 s?
   \[ F \Delta t = m \Delta v, \text{ so } F = \frac{m \Delta v}{\Delta t} = \frac{(0.25 \text{ kg})[(-14 \text{ m/s}) - (6.0 \text{ m/s})]}{2.0 \times 10^2 \text{ s}} = -2.5 \times 10^3 \text{ N} \]

9. A force of 1.21 x 10^3 N is needed to bring a car moving at 22.0 m/s to a halt in 20.0 s. What is the mass of the car?
   \[ m \Delta v = F \Delta t, \text{ so } m = \frac{F \Delta t}{\Delta v} = \frac{(-1.21 \times 10^3 \text{ N})(20.0 \text{ s})}{0 - 22.0 \text{ m/s}} = 1.0 \times 10^3 \text{ kg} \]

10. Small rockets are used to make small adjustments in the speed of satellites. One such rocket has a thrust of 35 N. If it is to be fired to change the velocity of a 72 000 kg spacecraft by 63 cm/s, how long should this rocket be fired?
    \[ F \Delta t, m \Delta v, \text{ so } \Delta t = \frac{m \Delta v}{F} = \frac{(72000 \text{ kg})(0.63 \text{ m/s})}{35 \text{ N}} = 1.3 \times 10^3 \text{ s} \]
    or about 22 min
11. A 10,000 kg freight car is rolling along a track at 3.0 m/s. Calculate the time needed for a force of 1000 N to stop the car.

\[ F \Delta t = m \Delta v, \text{ so } \Delta t = \frac{m \Delta v}{F} \]
\[ \Delta v = v_f - v_i = 0 - 3.00 \text{ m/s} \] and
\[ F = -1000 \text{ N} \] (the negative sign is because it is a retarding force), therefore
\[ \Delta t = \frac{(10,000 \text{ kg})(-3.00 \text{ m/s})}{(-1000 \text{ N})} = 30.0 \text{ s} \]

12. A car moving at 10 m/s crashes into a barrier and stops in 0.25 m.

a. Find the time required to stop the car.

\[ d = \frac{1}{2}(v_i + v_f)t, \text{ so} \]
\[ t = \frac{2d}{v_i + v_f} = \frac{2(0.25 \text{ m})}{0 + 10 \text{ m/s}} = 5.0 \times 10^{-2} \text{ s} \]

b. If a 20 kg child were to be stopped in the same time as the car, what average force must be exerted?

\[ F \Delta t = m \Delta v, \text{ so} \]
\[ F = \frac{m \Delta v}{\Delta t} = \frac{(20 \text{ kg})(0 - 10 \text{ m/s})}{5.0 \times 10^{-2} \text{ s}} \]
\[ = -4.0 \times 10^3 \text{ N} \]

c. Approximately what is the mass of an object whose weight equals the force from part b? Could you lift such a mass with your arm?

\[ W = mg, \text{ so} \]
\[ m = \frac{W}{g} = \frac{4.0 \times 10^3 \text{ N}}{9.80 \text{ m/s}^2} = 4.1 \times 10^2 \text{ kg} \]

No.

d. What does your answer to part c say about holding an infant on your lap instead of using a separate infant restraint?

Holding the child on your lap is dangerous to both the child and yourself.

13. An animal-rescue plane flying due east at 36.0 m/s drops a bale of hay from an altitude of 60.0 m. If the bale of hay weighs 175 N, what is the momentum of the bale the moment it strikes the ground?

First use projectile motion to find the velocity of the bale.
\[ v_x = 36.0 \text{ m/s} \]
\[ v_{y_0}^2 = v_y^2 + 2gd, \text{ so} \]
\[ v_y = \sqrt{2gd} = \sqrt{2(-60.0 \text{ m})(-9.80 \text{ m/s}^2)} \]
\[ = \sqrt{1.18 \times 10^3 \text{ m}^2/\text{s}^2} = 34.3 \text{ m/s} \]
\[ v = \sqrt{v_x^2 + v_y^2} = \sqrt{(36.0 \text{ m/s})^2 + (34.3 \text{ m/s})^2} \]
\[ = 49.7 \text{ m/s} \]

Now find the mass from \( w = mg, \) so
\[ m = \frac{W}{g} = \frac{175 \text{ N}}{9.80 \text{ m/s}^2} = 17.9 \text{ kg} \]
\[ mv = (17.9 \text{ kg})(49.7 \text{ m/s}) = 888 \text{ kg} \cdot \text{m/s} \]

Now the angle from the two velocities.
\[ \tan \theta = \frac{v_y}{v_x} = \frac{34.3 \text{ m/s}}{36.0 \text{ m/s}} \text{ so } \theta = 43.6^\circ \]

The momentum is 888 kg·m/s at 43.6° below horizontal.

14. A 10 kg lead brick falls from a height of 2.0 m.

a. Find its momentum as it reaches the ground.
\[ v_{y_0}^2 = v_{y_f}^2 + 2gd, \text{ so} \]
\[ v_{y_f} = \sqrt{2gd} = \sqrt{2(9.80 \text{ m/s}^2)(2.0 \text{ m})} = 6.3 \text{ m/s} \]
\[ mv = (10 \text{ kg})(6.3 \text{ m/s}) = 63 \text{ kg} \cdot \text{m/s} \]

b. What impulse is needed to bring the brick to rest?

\[ F \Delta t = m \Delta v = 63 \text{ N} \cdot \text{s} \]
c. The brick falls onto a carpet, 1.0 cm thick. Assuming the force stopping it is constant, find the average force the carpet exerts on the brick.

\[ \Delta t = \frac{2d}{v_{f} + v_{i}} = \frac{2(0.010 \text{ m})}{0 + 6.3 \text{ m/s}} = 3.2 \times 10^{-3} \text{ s} \]

\[ F \Delta t = m \Delta v, \text{ so} \]

\[ F = \frac{m \Delta v}{\Delta t} = \frac{63 \text{ kg} \cdot \text{m/s}}{3.2 \times 10^{-3} \text{ s}} = 2.0 \times 10^4 \text{ N} \]

d. If the brick falls onto a 5.0 cm foam rubber pad, what constant force is needed to bring it to rest?

\[ \Delta t = \frac{2d}{v_{f} + v_{i}} = \frac{2(0.050 \text{ m})}{0 + 6.3 \text{ m/s}} = 1.6 \times 10^{-2} \text{ s} \]

\[ F = \frac{m \Delta v}{\Delta t} = \frac{63 \text{ kg} \cdot \text{m/s}}{1.6 \times 10^{-2} \text{ s}} = 4.0 \times 10^3 \text{ N} \]

5. A 60 kg dancer leaps 0.32 m high.

a. With what momentum does the dancer reach the ground?

\[ v_f^2 = v_i^2 + 2gd, \text{ so} \]

\[ v_i = \sqrt{2gd} = \sqrt{2(9.80 \text{ m/s}^2)(0.32 \text{ m})} \]

\[ = 2.5 \text{ m/s} \]

\[ mv = (60 \text{ kg})(2.5 \text{ m/s}) = 1.5 \times 10^2 \text{ kg} \cdot \text{m/s} \]

b. What impulse is needed to make a stop?

\[ F \Delta t = m \Delta v = 1.5 \times 10^2 \text{ N} \cdot \text{s} \]

c. As the dancer lands, the knees bend, lengthening the time required to stop to 0.050 s. Find the average force exerted on the body.

\[ F \Delta t = 1.5 \times 10^2 \text{ N} \cdot \text{s}, \text{ so} \]

\[ F = \frac{1.5 \times 10^2 \text{ N} \cdot \text{s}}{\Delta t} = \frac{1.5 \times 10^2 \text{ N} \cdot \text{s}}{0.050 \text{ s}} \]

\[ = 3.0 \times 10^3 \text{ N} \]

d. Compare the stopping force to the performer's weight.

\[ W = mg = (60 \text{ kg})(9.80 \text{ m/s}^2) \]

\[ = 5.9 \times 10^2 \text{ N} \]

or the force is about 5 times the weight.

16. A fullback with a mass of 95 kg and running at 8.2 m/s collides in midair with a 128 kg defensive tackle moving in the opposite direction. Both players end up with zero speed.

a. What was the fullback's momentum before the collision?

\[ mv = (95 \text{ kg})(8.2 \text{ m/s}) = 7.8 \times 10^2 \text{ kg} \cdot \text{m/s} \]

b. What was the change in the fullback's momentum?

\[ 0 - 7.8 \times 10^2 \text{ kg} \cdot \text{m/s} \]

\[ = -7.8 \times 10^2 \text{ kg} \cdot \text{m/s} \]

c. What was the change in the tackle's momentum?

\[ 7.8 \times 10^2 \text{ kg} \cdot \text{m/s} \]

d. What was the tackle's original momentum?

\[ 7.8 \times 10^2 \text{ kg} \cdot \text{m/s} \]

e. How fast was the tackle moving originally?

\[ mv = 7.8 \times 10^2 \text{ kg} \cdot \text{m/s}, \text{ so} \]

\[ m = \frac{7.8 \times 10^2 \text{ kg} \cdot \text{m/s}}{128 \text{ kg}} = 6.1 \text{ m/s} \]

17. A glass ball, ball A, of mass 5.0 g moves at a velocity of 20.0 cm/s. It collides with a second glass ball, ball B, of mass 10.0 g moving along the same line with a velocity of 10.0 cm/s. After the collision, ball A is moving with a velocity of 8.0 cm/s.

a. What was the original momentum of ball A?

\[ m_A v_A = (5.0 \times 10^{-3} \text{ kg})(0.200 \text{ m/s}) \]

\[ = 1.0 \times 10^{-3} \text{ kg} \cdot \text{m/s} \]

b. What is the change in momentum of ball A?

\[ m_A \Delta v_A \]

\[ = (5.0 \times 10^{-3} \text{ kg})(0.080 \text{ m/s} - 0.200 \text{ m/s}) \]

\[ = -6.0 \times 10^{-4} \text{ kg} \cdot \text{m/s} \]

c. What is the change in momentum of ball B?

\[ +6.0 \times 10^{-4} \text{ kg} \cdot \text{m/s} \]
d. What is the momentum of ball \( B \) after the collision?

\[
m_{v_f} = m_{v_i} + \Delta m v = (10.0 \times 10^{-3} \text{ kg})(0.100 \text{ m/s}) + 6.0 \times 10^{-4} \text{ kg}\cdot\text{m/s} = 1.60 \times 10^{-2} \text{ kg}\cdot\text{m/s}
\]

e. What is ball \( B \)'s speed after the collision?

\[
m v = 1.60 \times 10^{-2} \text{ kg}\cdot\text{m/s}, \text{ so}
\]

\[
v = \frac{1.6 \times 10^{-3} \text{ kg}\cdot\text{m/s}}{10.0 \times 10^{-3} \text{ kg}} = 0.160 \text{ m/s} = 16.0 \text{ cm/s}
\]

18. Before a collision, a 25 kg object is moving at 12 m/s. Find the impulse that acted on this object if after the collision

a. it moves at 8.0 m/s.

\[
F \Delta t = m \Delta v = (25 \text{ kg})(8.0 \text{ m/s} - 12 \text{ m/s}) = -1.0 \times 10^2 \text{ kg}\cdot\text{m/s}
\]

b. it moves at -8.0 m/s.

\[
F \Delta t = m \Delta v = (25 \text{ kg})(-8.0 \text{ m/s} - 12 \text{ m/s}) = -5.0 \times 10^2 \text{ kg}\cdot\text{m/s}.
\]

19. A 2575 kg van runs into the back of a 825 kg compact car at rest. They move off together at 8.5 m/s. Assuming no friction with the ground, find the initial speed of the van.

\[
p_{A} + p_{B} = p_{A}' + p_{B}'
\]

\[
m_{A}v_{A} = (m_{A} + m_{B})v', \text{ so}
\]

\[
v_{A} = \frac{(m_{A} + m_{B})v'}{m_{A}} = \frac{(2575 \text{ kg} + 825 \text{ kg})(8.5 \text{ m/s})}{2575 \text{ kg}} = 11 \text{ m/s}
\]

20. A 15 g bullet is shot into a 5085 g wooden block standing on a frictionless surface. The block, with the bullet in it, acquires a velocity of 1.0 m/s. Calculate the velocity of the bullet before striking the block.

\[
m_{b}v_{b} + m_{w}v_{w} = (m_{b} + m_{w})v_{i} \text{ if } v_{w} = 0,
\]

\[
v_{b} = \frac{(m_{b} + m_{w})v}{m_{b}} = \frac{(15 \text{ g} + 5085 \text{ g})(1.0 \text{ m/s})}{15 \text{ g}} = 3.4 \times 10^2 \text{ m/s}
\]

21. A hockey puck, mass 0.115 kg, moving at 35.0 m/s, slides into a boot thrown on the ice by a fan. The boot has a mass of 0.265 kg. The puck and boot slide off together. Find their velocity.

\[
m_{p}v_{p} + m_{b}v_{b} = (m_{p} + m_{b})v', \text{ so}
\]

\[
v' = \frac{m_{p}v_{p} + m_{b}v_{b}}{m_{p} + m_{b}} = \frac{(0.115 \text{ kg})(35 \text{ m/s})}{0.115 \text{ kg} + 0.265 \text{ kg}} = 10.6 \text{ m/s}
\]

22. A 50 kg woman is riding on a 10 kg cart and they are moving east at 5.0 m/s. The woman jumps off the cart and hits the ground at 7.0 m/s eastward, relative to the ground. Calculate the velocity of the cart after she jumps.

Let east be positive.

\[(m_{w} + m_{c})v = m_{w}v_{w} + m_{c}v_{c}, \text{ so}
\]

\[
v_{c} = \frac{[m_{w} + m_{c}]v - m_{w}v_{w}}{m_{c}} = \frac{[50 \text{ kg} + 10 \text{ kg}](5.0 \text{ m/s}) - (50 \text{ kg})(7.0 \text{ m/s})}{10 \text{ kg}} = -5.0 \text{ m/s or 5.0 m/s, west}
\]
23. Two students on roller skates stand face-to-face, then push each other away. One student has a mass of 90 kg, the other 60 kg. Find the ratio of their velocities just after their hands lose contact. Which student has the greater speed?

\[
P_A' + P_B' = 0 = P_A' + P_B', \text{ so } m_Av_A' + m_Bv_B' = 0, \text{ and } m_Av_A' = -m_Bv_B' \\
v_A' = \frac{-m_Bv_B'}{m_A} = \frac{-90}{60} = -1.5
\]

The negative sign shows that the velocities are in opposite directions. The student with the smaller mass has the larger velocity.

24. A car with mass 1245 kg moving at 29 m/s strikes a 2175 kg car at rest. If the two cars stick together, with what speed do they move?

\[
P_A + P_B = P_A' + P_B' \\
m_Av_A + m_Bv_B = m_Av_A' + m_Bv_B' = (m_A + m_B)v' \\
v' = \frac{m_Av_A}{m_A + m_B} = \frac{(1245 \text{ kg})(29 \text{ m/s})}{1245 \text{ kg} + 2175 \text{ kg}} = 11 \text{ m/s}
\]

25. A 92 kg fullback running at 5.0 m/s attempts to dive across the goal line for a touchdown. Just as he reaches the goal line, he is met head-on in midair by two 75 kg linebackers, one moving at 2.0 m/s and the other at 4.0 m/s. If upon collision they become entangled as one mass, with what velocity do they travel? Does the fullback score?

\[
P_A + P_B + P_C = P_A' + P_B' + P_C' \\
m_Av_A + m_Bv_B + m_Cv_C = m_Av_A' + m_Bv_B' + m_Cv_C' = (m_A + m_B + m_C)v' \\
v' = \frac{(m_Av_A + m_Bv_B + m_Cv_C)}{(m_A + m_B + m_C)} \\
= \frac{(92 \text{ kg})(5.0 \text{ m/s}) + (75 \text{ kg})(-2.0 \text{ m/s}) + (75 \text{ kg})(-4.0 \text{ m/s})}{92 \text{ kg} + 75 \text{ kg} + 75 \text{ kg}} \\
= 0.041 \text{ m/s}, \text{ over the goal line — touchdown!}
\]

26. A 5.00 g bullet is fired with a velocity of 100 m/s toward a 10 kg stationary solid block resting on a frictionless surface.

a. What is the change in momentum of the bullet if it becomes embedded in the block?

\[
m_Bv_B = m_Bv' + m_wv' = (m_B + m_w)v', \text{ so } v' = \frac{m_Bv_B}{m_B + m_w} = \frac{(5.00 \times 10^{-3} \text{ kg})(100 \text{ m/s})}{5.00 \times 10^{-3} \text{ kg} + 10.00 \text{ kg}} \\
= 5.0 \times 10^{-2} \text{ m/s}
\]

\[
\Delta m v = m_B(v' - v) = (5.00 \times 10^{-3} \text{ kg})(5.0 \times 10^{-2} \text{ m/s} - 100 \text{ m/s}) = -0.500 \text{ kg} \cdot \text{m/s}
\]

b. What is the change in momentum of the bullet if it ricochets in the opposite direction with a speed of 99 m/s — almost the same speed as it had originally?

\[
\Delta m v = m_B(v' - v) = (5.00 \times 10^{-3} \text{ kg})(-99 \text{ m/s} - 100 \text{ m/s}) = -0.995 \text{ kg} \cdot \text{m/s}
\]
27. A plastic ball of mass 0.200 kg moves with a velocity of 0.30 m/s. It collides with a second plastic ball of mass 0.100 kg moving along the same line at a velocity of 0.10 m/s. After the collision, the velocity of the 0.200 kg ball is 0.26 m/s. What is the new velocity of the first ball?

\[
m_A v'_A + m_B v'_B = m_A v_A + m_B v_B, \text{ so}
\]

\[
v'_A = \frac{m_A v_A + m_B v_B - m_B v_B'}{m_A}
\]

\[
= \frac{(0.200 \text{ kg})(0.30 \text{ m/s}) + (0.100 \text{ kg})(0.10 \text{ m/s}) - (0.100 \text{ kg})(0.26 \text{ m/s})}{0.200 \text{ kg}}
\]

\[
= 0.22 \text{ m/s in the original direction.}
\]

28. Figure 9-18 shows a brick weighing 24.5 N being released from rest on a 1.00 m, frictionless plane inclined at an angle of 30.0°. The brick slides down the incline and strikes a second brick weighing 36.8 N.

a. Calculate the speed of the brick at the bottom of the incline.

\[
F_{||} = F_w \sin \theta = (24.5 \text{ N})(\sin 30.0^\circ) = 12.3 \text{ N}
\]

\[
F = ma, \text{ and } m = \frac{W}{g} \text{ so } a = \frac{F}{m} = \frac{F g}{W} = \frac{F g}{F_w} = \frac{(12.3 \text{ N})(9.80 \text{ m/s}^2)}{24.5 \text{ N}} = 4.9 \text{ m/s}^2
\]

\[
v^2 = 2ad = 2(4.9 \text{ m/s}^2)(1.00 \text{ m}), \text{ so } v = 3.1 \text{ m/s}
\]

b. If the two bricks stick together, with what initial speed will they move along the table?

\[
m_A v_A = (m_A + m_B)v', \text{ so } v' = \frac{m_A v_A}{m_A + m_B} = \frac{(2.50 \text{ kg})(3.1 \text{ m/s})}{2.50 \text{ kg} + 3.76 \text{ kg}} = 1.24 \text{ m/s}
\]

c. If the force of friction acting on the two bricks is 5.0 N, how much time will elapse before the bricks come to rest?

\[
F \Delta t = m \Delta v, \text{ so } \Delta t = \frac{m \Delta v}{F} = \frac{(2.50 \text{ kg} + 3.76 \text{ kg})(1.24 \text{ m/s})}{5.0 \text{ N}} = 1.6 \text{ s}
\]

d. How far will the two bricks slide before coming to rest?

\[
d = \frac{1}{2}(v_i + v)t = \frac{1}{2}(1.24 \text{ m/s} + 0)(1.6 \text{ s}) = 0.99 \text{ m}
\]
29. Ball A rolling west at 3.0 m/s has a mass of 1.0 kg. Ball B has a mass of 2.0 kg and is stationary. After colliding with ball B, ball A moves south at 2.0 m/s. Calculate the momentum and velocity of ball B after the collision.

Before

\[ \begin{align*}
\text{B} & \quad \vec{v}_A \\
\text{A} & \quad \vec{v}_B
\end{align*} \]

After

\[ \begin{align*}
\text{B} & \quad \vec{v}'_B \\
\text{A} & \quad \vec{v}'_A
\end{align*} \]

Horizontal: \( m_A \vec{v}_A = m_B \vec{v}_B \), so
\[ m_A \vec{v}_A = (1.0 \text{ kg})(3.0 \text{ m/s}) = 3.0 \text{ kg} \cdot \text{m/s} \]
Vertical: \( 0 = m_A \vec{v}'_A + m_B \vec{v}'_B \), so
\[ m_A \vec{v}'_B = -(1.0 \text{ kg})(2.0 \text{ m/s}) = -2.0 \text{ kg} \cdot \text{m/s} \]

The vector sum is;
\[ \vec{v}' = \sqrt{(3.0 \text{ kg} \cdot \text{m/s})^2 + (2.0 \text{ kg} \cdot \text{m/s})^2} = 3.6 \text{ kg} \cdot \text{m/s} \]
\[ \tan \theta = \frac{2.0 \text{ kg} \cdot \text{m/s}}{3.0 \text{ kg} \cdot \text{m/s}} \Rightarrow \theta = 34^\circ \]

Therefore, \( m_B \vec{v}'_B = 3.6 \text{ kg} \cdot \text{m/s} \) at \( 34^\circ \) N of W
\[ \vec{v}' = \frac{3.6 \text{ kg} \cdot \text{m/s}}{2.0 \text{ kg}} = 1.8 \text{ m/s} \text{ at } 34^\circ \text{ N of W} \]

Horizontal: \( m_c \vec{v}_C = m_c \vec{v}_C' + m_B \vec{v}'_B \), so
\[ 7.0 \text{ N} \cdot \text{s} = m_c \vec{v}_C + 2.0 \text{ N} \cdot \text{s}, \text{ so} \]
\[ m_c \vec{v}_C = 5.0 \text{ N} \cdot \text{s} \]
Vertical: \( 0 = m_c \vec{v}_C' + m_B \vec{v}'_B \), so
\[ m_c \vec{v}'_C = -2.0 \text{ N} \cdot \text{s} \]
The vector sum is;
\[ \sqrt{(5.0 \text{ N} \cdot \text{s})^2 + (2.0 \text{ N} \cdot \text{s})^2} = 5.4 \text{ N} \cdot \text{s} \]
\[ \tan \theta = \frac{2.0 \text{ N} \cdot \text{s}}{5.0 \text{ N} \cdot \text{s}} \Rightarrow \theta = 22^\circ \]

Therefore \( m_c \vec{v}'_C = 5.4 \text{ N} \cdot \text{s} \) at \( 22^\circ \) from original direction.

31. A space probe of mass 7600 kg is traveling through space at 120 m/s. Mission control determines that a change in course of 30.0° is necessary and, by electronic communication, instructs the probe to fire rockets perpendicular to its direction of motion. If the escaping gas leaves the craft's rockets at an average speed of 3200 m/s, what mass of gas should be expelled?

\[ \tan 30.0^\circ = \frac{m \Delta v}{m_p \Delta v_p} \]
\[ m \Delta v = m_p \Delta v_p \tan 30.0^\circ \]
\[ = (7600 \text{ kg})(120 \text{ m/s})(\tan 30.0^\circ) \]
\[ = 5.3 \times 10^4 \text{ kg} \cdot \text{m/s} \]
\[ m_p \Delta v_p = m_p \Delta v_p = m \Delta v \]
\[ m = m \frac{\Delta v}{\Delta v_p} = (5.3 \times 10^4 \text{ kg} \cdot \text{m/s}) \]
\[ = (3.2 \times 10^3 \text{ m/s}) \]
\[ = 170 \text{ kg} \]
32. Figure 9–19, which is drawn to scale, shows two balls during an elastic collision. The balls enter from the left of the page, collide, and bounce away. The heavier ball at the top of the diagram has a mass of 600 g, while the ball on the bottom has a mass of 400 g. Using a vector diagram, determine if momentum is conserved in this collision. Hint: Remember that the two masses are not equal. Try to account for any discrepancy found in the total momentum before and after the collision.

\[ \frac{F_n}{\Delta t} = -\frac{m_1 \Delta v_1}{\Delta t} \]

Dotted lines show that the changes of momentum for each ball are equal and opposite: \( \Delta (m_1 v_1) = \Delta (m_2 v_2) \). Since the masses are in a 3:2 ratio, a 2:3 ratio of velocities will compensate.

33. The head of a hammer, mass 1.0 kg, moving at 3.6 m/s, strikes a nail and drives it into hardwood.

a. The head stays in contact 2.0 ms and rebounds with negligible velocity. What is the average force exerted on the nail?

The force on the nail is opposite the force on the hammer, so

\[ F_n = -\frac{m_1 \Delta v_1}{\Delta t} \]

\[ = -\frac{(1.0 \text{ kg})(0 - 3.6 \text{ m/s})}{2.0 \times 10^{-3} \text{ s}} \]

\[ = 1.8 \times 10^3 \text{ N} \]

b. When the same hammer hits a springy nail, it rebounds with the same speed, 3.6 m/s. The contact time is the same. What force is exerted this time?

\[ F_n = -\frac{m_1 \Delta v_1}{\Delta t} \]

\[ = -\frac{(1.0 \text{ kg})(3.6 \text{ m/s} - 3.6 \text{ m/s})}{2.0 \times 10^{-3} \text{ s}} \]

\[ = 3.6 \times 10^3 \text{ N} \]

Supplemental Problems (Appendix B)

1. Jim strikes a 0.058-kg golf ball with a force of 272 N and gives it a velocity of 62.0 m/s. How long was the club in contact with the ball?

\[ \Delta t = \frac{m \Delta v}{F} = \frac{(0.058 \text{ kg})(62.0 \text{ m/s})}{272 \text{ N}} = 0.013 \text{ s} \]

2. A force of 186 N acts on a 7.3-kg bowling ball for 0.40 s.

a. What is the bowling ball's change in momentum?

\[ \Delta p = F \Delta t = (186 \text{ N})(0.40 \text{ s}) = 74 \text{ N} \cdot \text{s} \]

b. What is its change in velocity?

\[ \Delta v = \frac{\Delta p}{M} = \frac{74 \text{ N} \cdot \text{s}}{7.3 \text{ kg}} = 1.0 \times 10^1 \text{ m/s} \]

3. A 5500-kg freight truck accelerations from 4.2 m/s to 7.8 m/s in 15.0 s by applying a constant force.

a. What change in momentum occurs?

\[ \Delta p = m \Delta v = (5500 \text{ kg})(7.8 \text{ m/s} - 4.2 \text{ m/s}) \]

\[ = 2.0 \times 10^4 \text{ kg} \cdot \text{m/s} \]

b. How large of a force is exerted?

\[ F = \frac{\Delta p}{\Delta t} = \frac{2.0 \times 10^4 \text{ kg} \cdot \text{m/s}}{15.0 \text{ s}} \]

\[ = 1.3 \times 10^3 \text{ N} \]

and Its Conservation
4. In running a ballistics test at the police department, Officer Spears fires a 6.0-g bullet at 350 m/s into a container that stops it in 0.30 m. What average force stops the bullet?

\[ \Delta P = m \Delta v = (0.0060 \text{ kg})(-350 \text{ m/s}) = -2.1 \text{ kg} \cdot \text{m/s} \]

\[ t = \frac{2}{v_f + v_i} = (0.30 \text{ m}) \left[ \frac{2}{0 \text{ m/s} + 350 \text{ m/s}} \right] = 1.7 \times 10^{-3} \text{ s} \]

\[ F = \frac{\Delta P}{\Delta t} = \frac{-2.1 \text{ kg} \cdot \text{m/s}}{1.7 \times 10^{-3} \text{ s}} = -1.2 \times 10^3 \text{ N} \]

5. A 0.24-kg volleyball approaches Jennifer with a velocity of 3.8 m/s. Jennifer bumps the ball giving it a velocity of -2.4 m/s. What average force did she apply if the interaction time between her hands and the ball is 0.025 s?

\[ F = \frac{m \Delta v}{\Delta t} = \frac{(0.24 \text{ kg})(-2.4 \text{ m/s} - 3.8 \text{ m/s})}{0.025 \text{ s}} = -6.0 \times 10^1 \text{ N} \]

6. A 0.145-kg baseball is pitched at 42 m/s. The batter hits it horizontally to the pitcher at 58 m/s.

   a. Find the change in momentum of the ball.

   Take the direction of the pitch to be positive direction
   \[ \Delta P = mv_f - mv_i = m(v_f - v_i) = (0.145 \text{ kg})(-58 \text{ m/s} - (+42 \text{ m/s})) = -14.5 \text{ kg} \cdot \text{m/s} \]

   b. If the ball and bat were in contact 4.6 \times 10^{-4} \text{ s}, what would be the average force while they touched?

   \[ F \Delta t = \Delta P, \]
   \[ F = \frac{\Delta P}{\Delta t} = \frac{-14.5 \text{ kg} \cdot \text{m/s}}{(4.6 \times 10^{-4} \text{ s})} = -3.2 \times 10^4 \text{ N} \]

7. A 550-kg car traveling at 24.0 m/s collides head-on with a 680-kg pick-up truck. Both vehicles come to a complete stop upon impact. Use car's direction as positive.

   a. What is the momentum of the car before collision?

   \[ P = mv = (550 \text{ kg})(24.0 \text{ m/s}) = 1.32 \times 10^4 \text{ kg} \cdot \text{m/s} \]

   b. What is the change in the car's momentum?

   \[ -1.32 \times 10^4 \text{ kg} \cdot \text{m/s}, \text{ since car stops on impact} \]

   c. What is the change in the truck's momentum?

   \[ +1.32 \times 10^4 \text{ kg} \cdot \text{m/s}, \text{ by conservation of momentum} \]

   d. What is the velocity of the truck before collision?

   \[ v = \frac{P}{M} = \frac{-1.32 \times 10^4 \text{ kg} \cdot \text{m/s}}{680 \text{ kg}} = -19.4 \text{ m/s} \]
8. A truck weighs four times as much as a car. If the truck coasts into the car at 12 km/h and they stick together, what is their final velocity?

Momentum before: \((4 \text{ m})(12 \text{ km/h})\).
Momentum after: \((4m + 1m)v\), so
\[v = (4m/5m)(12 \text{ km/h}) = 9.6 \text{ km/h}\]

9. A 50.0– projectile is launched with a horizontal velocity of 647 m/s from a 4.65–kg launcher moving in the same direction at 2.00 m/s. What is the velocity of the launcher after the projectile is launched?

\[p_A + p_b = p_A' + p_b',\]
\[m_Av_A + m_Bv_B = m_Av_A' + m_Bv_B',\]
so \[v_B' = (m_Av_A + m_Bv_B - m_Av_A')/m_B.\]
Assuming projectile \((A)\) is launched in direction of launcher \((B)\) motion,
\[v_B' = (0.0500 \text{ kg})(2.00 \text{ m/s}) + (4.65 \text{ kg})(2.00 \text{ m/s}) - (0.0500 \text{ kg})(647 \text{ m/s}) \]
\[= -4.65 \text{ kg}, \text{ or } 4.94 \text{ m/s backwards}\]

10. Two lab carts are pushed together with a spring mechanism compressed between them. Upon release, the 5.0–kg cart repels one way with a velocity of 0.12 m/s while the 2.0–kg cart goes in the opposite direction. What velocity does it have?

\[m_1v_1 = m_2v_2\]
\[(5.0 \text{ kg})(0.12 \text{ m/s}) = (2.0 \text{ kg})(v_2)\]
\[v_2 = 0.30 \text{ m/s}\]

11. A 12.0–kg rubber bullet travels at a velocity of 150 m/s, hits a stationary 8.5–kg concrete block resting on a frictionless surface, and ricochets in the opposite direction with a velocity of \(-100 \text{ m/s}\). How fast will the concrete block be moving?

Momentum of bullet before collision: \(P_B = (0.0120 \text{ kg})(150 \text{ m/s}) = 1.80 \text{ kg} \cdot \text{m/s}\)
Momentum of bullet and block after collision: \(P_A = (0.0120 \text{ kg})(-100 \text{ m/s}) + (8.5 \text{ kg})(v)\)
\[P_B = P_A\]
\[1.80 \text{ kg} \cdot \text{m/s} = (0.0120 \text{ kg})(-100 \text{ m/s}) + (8.5 \text{ kg})(v)\]
\[v = \frac{3.0 \text{ kg} \cdot \text{m/s}}{8.5 \text{ kg}} = 0.35 \text{ m/s}\]

12. A 6500–kg freight car traveling at 2.5 m/s collides with an 8000–kg stationary freight car. If they interlock upon collision, what is their velocity?

\[m_1v_1 + m_2v_2 = (m_1 + m_2)v\]
\[(6500 \text{ kg})(2.5 \text{ m/s}) = (6500 \text{ kg} + 8000 \text{ kg})v\]
\[v = 1.1 \text{ mfs}\]

13. Tim, mass 42.00 kg, is riding a skateboard, mass 2.00 kg, traveling at 1.20 m/s. Tim jumps off and the skateboard stops dead in its tracks. In what direction and with what velocity did he jump?

\[(m_1 + m_2)v_b = m_1v_{A} + m_2v\]
\[(42.00 \text{ kg} + 2.00 \text{ kg})(1.20 \text{ m/s}) = (42.00 \text{ kg})(v_A) + 0\]
\[v_A = 1.26 \text{ m/s in the same direction as he was riding.}\]
14. A cue ball, mass 0.16 kg, rolling at 4.0 m/s, hits a stationary eight-ball of similar mass. If the cue ball travels 45° above its original path, and the eight-ball at 45° below, what is the velocity of each after collision?

\[ P_{\text{before}} = (0.16 \text{ kg})(4.0 \text{ m/s}) = 0.64 \text{ kg} \cdot \text{m/s} \]
\[ P_8 = P_C \]
\[ \cos 45^\circ = \frac{P_8}{0.64 \text{ kg} \cdot \text{m/s}} \]
\[ P_8 = 0.45 \text{ kg} \cdot \text{m/s} \]
\[ \nu = \frac{0.45 \text{ kg} \cdot \text{m/s}}{0.16 \text{ kg}} = 2.8 \text{ m/s} \]

15. Two opposing hockey players, one of mass 82.0 kg skating north at 6.0 m/s and the other of mass 70.0 kg skating east at 3.0 m/s, collide and become tangled.

a. Draw a vector-momentum diagram of the collision.

b. In what direction and with what velocity do they move after collision?

\[ \tan \theta = \frac{492}{210} = 67^\circ \]
\[ R^2 = (492^2 + 210^2) \text{kg}^2 \cdot \text{m}^2/\text{s}^2 \]
\[ R = 535 \text{ kg} \cdot \text{m/s} \]
\[ \nu = \frac{535 \text{ kg} \cdot \text{m/s}}{152 \text{ kg}} = 3.5 \text{ m/s, 67}^\circ \]